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A New General Perturbation Method for Determining the Long-Term Motion of Comets

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Abstract: A number of authors have used special perturbation methods to propagate Comet Halley back before its oldest observation in 239 BC. Unfortunately, results from these studies vary drastically because it is so difficult to accurately model nongraviatational forces acting on comets. In contrast, general perturbation methods do not need to model any forces and can be more accurate over long periods of time. Regrettably, the most recent general perturbation method used for Comet Halley introduced a lot of subjectivity. A new general perturbation method integrating Halley's Comet back in time is presented here. This new method uses least squares, based solely on math. Therefore, it does not introduce any subjectivity. It also permits statistical analysis of the model's accuracy. Using this model, Halley's Comet is propagated back to 2317 BC, and with the derived equations it can easily be integrated back much further in time. Results are very similar to two previous studies by other authors, varying by less than five years when propagated back over 2,200 years. This same new general perturbation method is also applied to Comet Swift-Tuttle. Results with Swift-Tuttle compare reasonably well with the only other known research that integrated this comet back in time.

Keywords: Comet, Halley, Swift-Tuttle, Orbit Determination, Perturbations

1. Introduction

In 1695, Edmond Halley wrote his good friend Isaac Newton a private letter. Referring to the comet of 1531, Halley stated, "I am more and more confirmed that we have seen that Commet now three times, since ye Yeare 1531" [1] Ten years later, Halley published the orbital elements for 24 observed comets and publically noted the similarities of the comets from 1682, 1607, and 1531. He claimed that they were actually the same comet and made the bold prediction "that it will return again in the year 1758. And posterity will not refuse to acknowledge that this was first discovered by an Englishman." [2] Sixteen years after his death, his prediction was verified, and the comet was named in his honor.

Since then, many others have tried to track Comet Halley back in time to link it with earlier observations. Pingré was able to determine the perihelion passage of the great comet of 1456 and confirm that this was an earlier passage of Halley's Comet. However, due to his rough determination of the orbits for the comets of 837 and 1301, Pingré failed to realize these were also passages of Halley. [3] Using just the average time between perihelion passages, Biot stepped Halley's Comet back in time to Chinese observations from 65 BC. [4] Given Halley's predictable orbit, this method of moving 76 years back at a time is reasonably accurate. This is how Hind initially linked Comet Halley to observations between 1301 and 11 BC. He then compared other orbital elements with Halley. In many cases, this method gave him correct answers, but he also incorrectly identified other passages. [5, 6]

Later, more sophisticated methods of stepping Halley back in time were used to estimate prior perihelion passages before any known observations. These techniques can be grouped into two broad categories: special perturbation and general perturbation methods. Both can be used to integrate a comet or satellite backward or forward in time, accounting for a range of forces acting on the comet. [7]

This paper reviews previous work integrating Halley's Comet back in time, including both special and general perturbation methods. Then a new general perturbation method is explained, which yields similar results as previous research. Finally, this new method is applied to Comet Swift-Tuttle with favorable results.

2. Previous Research

2.1. Special Perturbation Methods

Special perturbation methods numerically integrate the equations of motion acting on a comet forward or backward in time accounting for all perturbing forces. This is called a special perturbation method because it is only valid for the specific initial conditions and forces modelled.

All special perturbation methods work in the same way. Beginning at a known position, all of the forces acting on a comet are calculated. This includes the gravitational force from the sun, and for most advanced models, it also includes gravitational forces from planets as well as nongravitational forces caused by outgassing as the comet gets closer to the Sun. A computer program assumes these forces are constant for a small step in time and calculates the comet's future position and velocity after this small step. The computer program then moves the planets by that same time interval, and the whole process is repeated over and over, moving one small step at a time, either forward or backward in time.

Chang was one of the first to use this special perturbation method to propagate Halley's Comet back in time. He integrated it back to 1057, accounting for the gravitational forces of the Sun, Venus, Earth, Jupiter, Saturn, Uranus, and Neptune. He did not account for the other planets or any nongravitational forces. [8]

It is relatively simple for special perturbation methods to calculate the gravitational forces acting on comets. Because the locations of planets are well known, it is easy to calculate the forces they exert on a comet using Newton's law of gravity. However, it is much more difficult to determine the nongravitational forces caused by outgassing, and the accuracy of modelling these forces is generally the limiting factor for integrating comets back in time. [9]

In 1950, Whipple was one of the first to try to model the nongravitational forces acting on comets. [10] Later, between 1969 and 1973, Marsden, Sekanina, and Yeomans collectively published five other papers that also modelled these nongravitational forces. Their final model includes three parameters, which differ for each comet. [11] Using this model, Yeomans numerically integrated Halley's orbit from the 1682, 1759, and 1835-1836 observations back to 837 using a least squares differential correction. Yeoman's program showed Halley came within 0.04 AU of the Earth in 837. Because this close of a pass would significantly perturb the comet's orbit, no attempt was made to integrate Halley further back in time. [12] Fortunately, Yeomans and Kiang were later able to include ancient Chinese observations from 239 BC, which allowed them to integrate Halley back further in time. To link Halley with these ancient Chinese observations, they had to adjust the comet's computed perihelion time and eccentricity following the close pass by Earth in 837. Using a step size of 0.5 days, they accounted for gravitational forces from all the planets and assumed the parameters for the nongravitational force were constant. With these assumptions, they propagated Comet Halley back to 1403 BC. [6]

The following year, another group published a slightly different method to estimate the nongravitational forces acting on comets. [13] Landgraf used their model to propagate Halley's Comet. He also noticed that the nongravitational forces acting on Comet Halley increase about 0.5% each revolution, likely due to the comet's deceasing mass. Therefore, instead of using a model that kept nongravitational forces constant, as Yeomans and Kiang had done, Landgraf's model used a least squares correction that allowed the nongravitational parameters to vary linearly with time. With that model, he integrated Halley back to 2316 BC. [14, 15]

In contrast, Sitarski noted a parabolic time dependency in the nongravitational forces acting on Comet Halley. Therefore, his work modelled the nongravitational parameters as a parabolic function with time, and he integrated Comet Halley back to 1457 BC. [16-19]

It is interesting to note the different results of these three most recent special perturbation studies. Table 2 shows the perihelion times of Comet Halley (in years BC) for each of these studies beginning in 239 BC, the date of the last observed perihelion passage. The most significant difference is Yeomans and Kiang assumed the parameters modelling nongravitational forces were constant. Landgraf assumed those parameters had a linear relationship with time, and Sitarski assumed they had a parabolic relationship with time. Notice from Table 2, Landgraf's model calculated Halley passed perihelion in 1472.8 BC, which is 15 years different from Sitarski's calculations and almost 70 years different from Yeomans and Kiang's model! As others have noted, imperfectly modelled nongravitational forces are generally the limiting factor for the integrating comet orbits, and models of these forces change frequently. [9] This illustrates a drawback to special perturbation methods. If the forces cannot be calculated accurately, the model's errors grow significantly over time.

2.2. General Perturbation Methods

In contrast to special perturbation methods, which numerically integrate the position and velocity vectors forward or backward in time, general perturbation methods analytically integrate the orbital elements. This is advantageous because orbital elements do not change as quickly as position and velocity vectors. In addition, because general perturbations use analytical methods, it is not necessary to model all the perturbing forces. Instead, an orbital element such as eccentricity can be modelled as a Taylor series, as shown in Equation 1. If the derivatives of eccentricity are known, it does not really matter what specific forces are causing $\dot{e}$, $\ddot{e}$, etc.

$$e_t = e_o + \dot{e}t + \frac{\ddot{e}}{2!}t^2 + \frac{\dddot{e}}{3!}t^3 + \ldots \quad (1)$$

However, most perturbations, including both gravitational forces from the planets and nongravitational forces, have periodic effects on orbital elements. Therefore, it is more common to model these changes using sines and cosines, with a Fourier series, show in Equation 2.

$$e_t = e_o + \sum_{n=1}^{\infty}(a_n\, cosnt + \, b_n sinnt) \quad (2)$$

Assuming all the forces can be calculated, special perturbation methods are usually more accurate than general perturbation methods for relatively short time frames. In contrast, general perturbation methods can be more accurate if some forces are difficult to calculate (like the nongravitational force acting on comets) or if the integration is over longer periods. Over long time intervals, general perturbation methods tend to smooth out small changes; this is one reason early methods were called "methods of averages". [7] More sophisticated general perturbation methods, sometimes called "Variation of Parameters", are much more accurate and are credited with the famous discovery of Neptune in 1845. [20]

Early attempts to trace Comet Halley back in time were a very crude example of a general perturbation method. When Biot, and later Hind, stepped 76 years back in time, looking for observations of retrograde comets, they were both essentially using a general perturbation method without any derivatives shown in Equation 1 or sines and cosines in Equation 2. Instead, they were assuming the time between perihelion passages was constant and just used an average of the observed times between perihelions. [4-6]

Kamieński used a more accurate model to analytically integrate Halley back in time before any recorded observations. His model included three sine and cosine terms, but instead of using the exact form of Equation 2, he used the equivalent series shown below with only cosine terms, including phase shifts, where T_n is the time between perihelion passages for the n^{th} orbit.

$$T_n = a + \mathrm{b}\cos(cn + d) + ecos(fn + g) + hcos(in + j) \quad (3)$$

Kamieński selected the coefficients, *a-j,* by inspection to achieve the best model. [19, 21, 22] Because these values were not determined mathematically, his method introduced some subjectivity. However, as shown in Table 2, it did yield results similar to the special perturbation methods.

3. A New General Perturbation Method for Comet Halley

Building upon Kamieński work, a new general perturbation method was developed to integrate Comet Halley back in time. Similar to Kamieński's work, this method looks at the time between observed perihelion passages. There are 30 observed perihelion passages of Halley's Comet. The most recent was in 1986, and the oldest observation was in 239 BC. Therefore, there are 29 times between successive perihelion passages, which are plotted in Figure 1.

Notice, the time between successive perihelion observations, T_n, only varies by about five years, with a minimum time of 74.4 years and a maximum of 79.3 years. As expected, the periodic effects of perturbations cause T_n to oscillate slightly; so, modelling this with a Fourier series, as shown in Equation 3, is preferred.

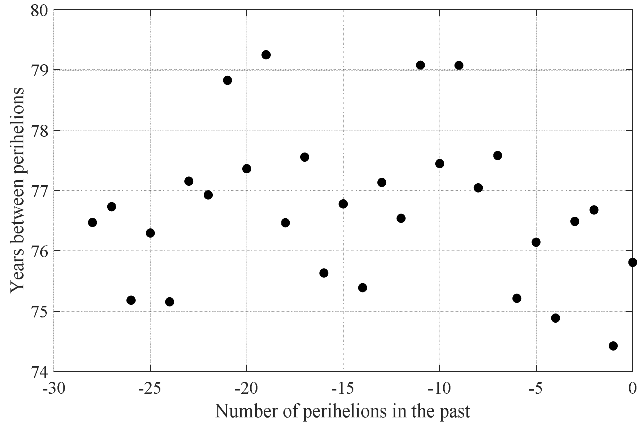

Figure 1. *Times between successive perihelion observations for Comet Halley. The x-axis show the number of orbits in the past. For example, the point on the far right shows the difference between the two most recent perihelion passages.*

Figure 2 shows the result of including just the first line of Equation 3, with only a constant and one cosine term. The solid line is the resulting model. Unlike Kamieński's method, however, the coefficients in this model were not determined subjectively, but were found mathematically using least squares. Two higher-order cosine terms were then added, as in Equation 3, to improve the model's accuracy. This resulted in the plot shown in Figure 3. A fourth cosine term was initially added, but it had an amplitude of zero, so it did not change the model at all and was therefore not included in the model.

All of the coefficients, *a-j* in Equation 3 were determined using least squares. Those values, shown in Table 1, resulted in a Root Mean Square (RMS) of 0.882 years.

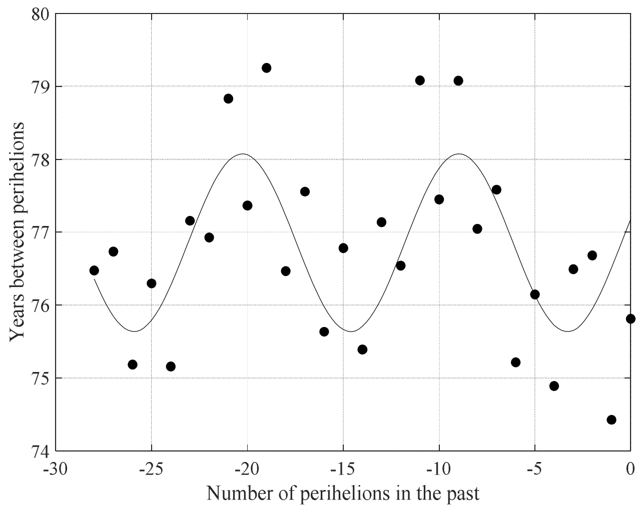

Figure 2. *A simplified model for the time between successive perihelion passages for Comet Halley. The solid circles indicate times between successive observed perihelion passages, and the solid line is the model using only the first periodic term from Equation 3,* $T_n = a + bcos(cn + d)$.

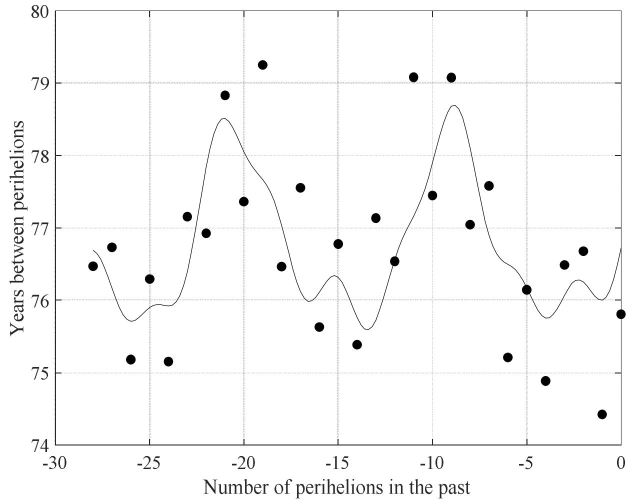

Figure 3. *A new model for the time between successive perihelion passages for Comet Halley. The solid circles indicate times between perihelion passages based on known observations, and the solid line is the best-fit model using all three periodic terms in Equation 3.*

Using least squares to determine the coefficients eliminates the subjectivity in Kamieński's work. It also permits statistical analysis of the model. The state, X, is defined as a matrix of the coefficients, *a-j*.

$$X = [a\ b\ c\ d\ e\ f\ g\ h\ i\ j]^T \tag{4}$$

The model, y_n, is given in Equation 3, and the A matrix is then found using partial derivatives.

$$A = \frac{\partial y}{\partial X} \tag{5}$$

Finally, the covariance matrix, P, is calculated from Equation 6, and the diagonal terms of P give the variance of the coefficients *a-j*. [7]

$$P = [A^T A]^{-1} \tag{6}$$

This allows the confidence in the coefficients *a-j* to be calculated. The coefficients for the model, along with their standard deviations found from P, are listed below in Table 1.

Using these values, a Monte Carlo simulation was conducted, varying the coefficients *a-j* in the model according their standard deviations. This allowed a one standard deviation line to be added to Figure 3, indicating the accuracy of this new model. This is shown in Figure 4.

Table 1. *Values and standard deviations of coefficients for a new model of the time between perihelion passages of Comet Halley, using Equation 3.*

Coefficient	Value	Standard Deviation
a	76.852910650	0.189945328
b	1.219831935	0.272152002
c	0.555673015	0.026694034
d	0.746092508	0.446938184
e	0.402349102	0.265249296
f	1.071608280	0.085542901
g	1.712073671	1.492714905
h	0.257794140	0.266285453
i	1.959332378	0.123671080
j	-0.108276329	2.084711338

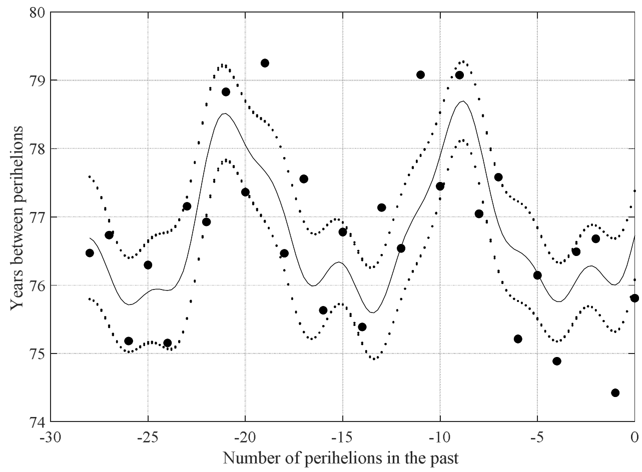

Figure 4. *A new model for the time between successive perihelion passages for Comet Halley with one standard deviation lines. The solid circles indicate times between perihelion passages based on known observations, and the solid line is the best-fit model. The dashed lines show the variability of the model within plus and minus one standard deviation.*

Once the coefficients *a-j* were calculated, it is very easy to calculate T_n as far back or forward in time as desired. Figure 5 shows those values for the last 56 orbits, 29 of which were observed, and the prior 27 orbits are predicted based on this new model. Using these times between successive perihelion passes, the passages prior to 239 BC were easily calculated, going back over 2,000 years, to 2317 BC. These dates are shown in the far right column of Table 2.

Notice the results from this new model are very similar to those of Landgraf and Kamieński, varying by an average of only 4.3 and 2.0 years respectively from the works of these two authors. However, there are larger differences between the work presented here and the work of Yeomans and Kiang (24.6 years on average) and Sitarski (9.1 years on average). These differences are most likely due to the methods they used to model the nongravitational forces acting on Comet Halley.

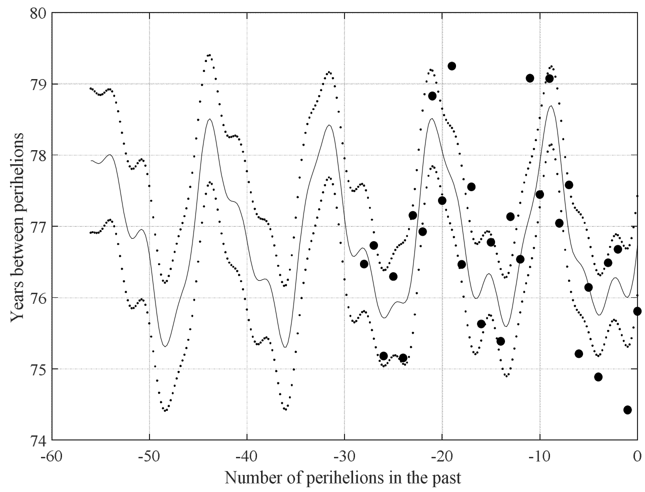

Figure 5. *The time between successive perihelion passages for Comet Halley extrapolated back 56 orbits, corresponding to 2317.6 BC. The circles indicate known times between perihelion passes. The solid line is the best-fit model, and the dashed lines show plus and minus one standard deviation.*

Table 2. *Perihelion passages (in years BC) for Comet Halley prior to 239 BC calculated by various authors.*

Special Perturbations			General Perturbations	
Yeomans and Kiang	**Landgraf**	**Sitarski**	**Kamieński**	**Brown**
239.6	239.7	239.8	238.7	239.8
314.3	314.6	314.9	313.8	316.3
390.3	390.7	391.1	390.7	393.4
465.5	465.7	466.1	466.7	471.6
539.6	541.0	542.7	544.9	550.0
615.4	617.3	619.2	622.0	627.8
689.9	692.0	694.1	701.1	705.2
762.4	768.9	770.2	778.2	781.4
835.6	845.6	845.6	856.6	856.7
910.6	923.9	922.4	932.8	932.4
985.1	1001.2	999.2	1009.9	1008.7
1058.1	1081.0	1075.0	1085.3	1085.0
1128.7	1158.5	1151.5	1162.3	1161.8
1197.6	1236.7	1226.7	1238.2	1239.1
1265.3	1315.7	1302.6	1316.4	1316.5
1333.4	1393.3	1379.6	1393.3	1394.6
1403.2	1472.8	1457.5	1472.3	1473.0
	1550.4		1549.6	1550.5
	1628.0		1627.8	1626.9
	1705.4		1704.1	1702.9
	1782.2		1781.0	1778.2
	1858.2		1856.2	1853.8
	1935.2		1932.6	1930.4
	2010.0		2008.3	2007.4
	2086.8		2083.7	2084.2
	2162.2		2160.3	2161.7
	2239.0		2235.6	2239.8
	2316.1		2312.9	2317.6

4. Appling this New Perturbation Method to Comet Swift-Tuttle

It is relatively easy to use this new general perturbation method to propagate other comets back in time if there is sufficient data to identify trends. Comet Swift-Tuttle is another periodic comet that has been observed many times and spends most of its time away from the plane of the ecliptic. Therefore, just like Halley, Comet Swift-Tuttle does not have large perturbations, and its orbit does not change significantly.

Swift-Tuttle has passed perihelion 20 times since 69 BC, resulting in 19 known times between perihelion passages. Using these observed data points, least squares was used to determine the best-fit model for the time between successive perihelions, T_n, using Equation 7 below.

$$T_n = a + \mathrm{b}\cos(cn + d) + ecos(fn + g) \tag{7}$$

Adding a third cosine term did not improve the model, so it was not included. This is likely because there are fewer data points with Comet Swift-Tuttle than there are for Halley. The resulting model had an RMS of 1.97 years, which as expected is slightly larger than it was for Comet Halley.

The coefficients, *a*-*g*, were found mathematically using least squares in the same manner as they were for Comet Halley. These values and their standard deviations are listed in Table 3, and the plot of T_n is shown in Figure 6.

Using the coefficients from Table 3, it is easy to calculate the times between perihelion passages prior to 69 BC. Using those times, the years of each perihelion were calculated. The five previous perihelion times (in years BC) were estimated to be 201.7, 334.3, 464.1, 590.2, and 714.8.

Table 3. Values and standard deviations of coefficients for the time between perihelion passages of Comet Swift-Tuttle using Equation 7.

Coefficient	Value	Standard Deviation
a	129.340891273	0.258891351
b	2.865508059	0.331685822
c	1.013625473	0.024005448
d	3.801243680	0.284222378
e	2.151617422	0.317320878
f	0.711866467	0.044131302
g	5.317172698	0.490163909

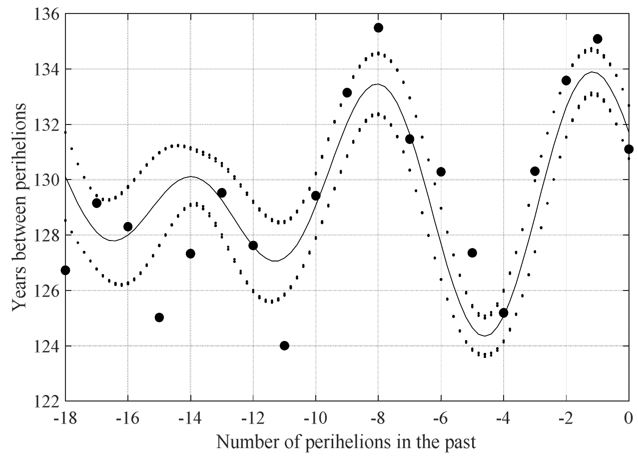

Figure 6. *The time between successive perihelion passages for Comet Swift-Tuttle. The solid circles indicate times between successive perihelion passages based on known observations, and the solid line is the best-fit model using Equation 7. The dashed lines show the variability of the model within plus and minus one standard deviation.*

These values vary slightly from previous work by Yau, Yeomans and Weissman, who used special perturbation methods without any nongravitational forces to trace Swift-Tuttle 634 years back in time from its last observation in 69 BC. They predicted the comet passed perihelion in 703 BC, a difference of 11 years from the model presented here. [23] Perhaps this difference is due to nongravitational forces that were not accounted for by Yau et al. or simply because there is not as much data available for Comet Swift-Tuttle as there is for Halley. However, this difference is similar to the different times the three special perturbation methods calculated for Comet Halley 634 years before its last observation, shown in Table 2.

5. Conclusion

Special perturbation methods are the most common way comet or satellite orbits are integrated forward or backward in time. However, because it is very difficult to accurately model every force acting on a comet, especially the nongravitational forces caused by outgassing, there can be significant differences between special perturbation models. In contrast, general perturbation methods do not need to calculate specific forces acting on a comet. Instead, general perturbation methods only require enough data to identify trends that are caused by those forces.

Kamieński used a general perturbation method, as did the research presented here. Both models used three cosine terms to estimate the small changes in time between successive perihelion passages of Halley's Comet. However, the new general perturbation model presented here improves upon Kamieński's work in two ways. Most importantly, instead of subjectively selecting coefficients to model the periodic changes in perihelion passages, the new model uses least squares to mathematically determine the best value for each coefficient. In addition, the least squares method allows the variance of each coefficient to be calculated, which gives a measure of accuracy for the model.

The new model developed for Comet Halley resulted in an RMS of only 0.882 years over the 2,200 years of observed perihelion passages. Also, when Halley was integrated back from 239 BC to 2317 BC, this new model yielded very similar results to Kamieński's work as well as the special perturbation model developed by Landgraf. Compared to their research, the new model developed here only varied by an average 2.0 and 4.3 years respectively over 2,000 years.

To demonstrate how this general perturbation method can easily be adapted to another comet, the same approach was taken with Comet Swift-Tuttle. Due to limited data, only two cosine terms were included, yet the results going back 634 years before the oldest perihelion observation were reasonably close to the propagation by Yau et al. using a special perturbation method.

In the future, the general perturbation method shown here could be improved by weighting observations differently based upon the confidence of each observation. It might also be improved by modelling parameters other than T_n, the time between successive perihelion passages. While it was convenient to use T_n for this research because it does not vary significantly, other parameters would permit more data to be used. For example, the semimajor axis changes slightly throughout a comet's orbit, but there are many measurements taken during each pass. This would result in many more data points, which could translate into a more accurate model.

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